

DEEP GALAXY COUNTS IN THE K BAND WITH THE KECK TELESCOPE¹

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Received 1994 August 3; accepted 1994 October 18

ABSTRACT

We present deep galaxy counts in the K ($\lambda 2.2 \mu\text{m}$) band, obtained at the W. M. Keck 10 m telescope. The data reach limiting magnitudes $K \sim 24$ mag, about 5 times deeper than the deepest published K -band images to date. The counts are performed in three small ($\sim 1'$), widely separated high-latitude fields. Extensive Monte Carlo tests were used to derive the completeness corrections and minimize photometric biases. The counts continue to rise, with no sign of a turnover, down to the limits of our data, with the logarithmic slope of $d \log N/dm = 0.315 \pm 0.02$ between $K = 20$ and 24 mag. This implies a cumulative surface density of $\sim 5 \times 10^5$ galaxies deg^{-2} , or $\sim 2 \times 10^{10}$ over the entire sky, down to $K = 24$ mag. Our counts are in good agreement with, although slightly lower than, those from the Hawaii Deep Survey by Cowie and collaborators; the discrepancies may be due to the small differences in the aperture corrections. The observed field-to-field variations are as expected from the Poissonian noise and galaxy clustering as described by the angular two-point correlation function for faint galaxies. We compare our counts with some of the available theoretical predictions. The data do not require models with a high value of Ω_0 , but can be well fitted by models with no (or little) evolution, and cosmologies with a low value of Ω_0 . Given the uncertainties in the models, it may be premature to put useful constraints on the value of Ω_0 from the counts alone. Optical-to-IR colors are computed, using CCD data obtained previously at Palomar. We find a few red galaxies with $(r - K) \gtrsim 5$ mag, or $(i - K) \gtrsim 5$ mag; these may be ellipticals at $z \sim 1$. While the redshift distribution of galaxies in our counts is still unknown, the flux limits reached would allow us to detect unobscured L_* galaxies out to substantial redshifts ($z > 3?$).

Subject headings: cosmology: observations — galaxies: evolution — galaxies: photometry

1. INTRODUCTION

Deep galaxy counts as a function of magnitude are one of the classical cosmological tests. Counts reflect the global geometry by tracing the comoving volume, and thus can in principle be used to constrain both the density parameter Ω_0 and the cosmological constant Λ_0 . However, they are also sensitive to the rates of galaxy evolution, since higher star formation rates in the past would make more distant galaxies detectable, and to galaxy merging rates, which would decrease the comoving number density of objects while increasing their luminosities. Galaxy counts in the K band are particularly valuable: they are less sensitive to the flicker in star formation rates and dust extinction, which can seriously affect the observed counts in the bluer bands. The K -corrections are also much better behaved in the K band than, for example, in the B band. Good reviews of the subject include those by Koo & Kron (1992), Ellis (1993), and Cowie & Songaila (1993), and references therein.

Deep galaxy counts in the K band have been described, for example, by Gardner et al. (1993), or Cowie et al. (1994). The W. M. Keck telescope provides us with an excellent opportunity to check independently, and to extend these observations to fainter magnitude levels. In an earlier paper (Soifer et al. 1994) we presented a first look at the deep K -band

galaxy counts with the Keck, using targeted observations in several fields. Here we present new, deeper observations of three “blank” fields, reaching to $K \sim 23.5$ –24 mag.

2. OBSERVATIONS AND DATA REDUCTIONS

The infrared observations were made during 5 nights between UT 1994 April 30 and 1994 May 7 with the W. M. Keck 10 m telescope on Mauna Kea. The near-infrared camera (NIRC) is based on a 256×256 pixel In:Sb array, with a pixel size of $0''.15$, and a full field of view of $38''.4 \times 38''.4$ (Matthews & Soifer 1994). We used a standard K filter and had typical seeing FWHM $\sim 0''.75$. We have obtained approximately 20,000 s total integration time using multiple short exposures (typically either sums of 12×10 s, or 6×20 s) on each of three high-latitude “blank” fields of sky, selected from our previous studies; infrared images are shown in Figure 1 (Plate 3). The telescope pointing was shifted by several arcseconds between each short exposure. Each field was chosen to include only one moderately bright star (~ 18 mag), to be used for registration of the images. For one field (FP1254), the stacking object turned out to be a galaxy. All nights were photometric, and we observed several faint standards Casali & Hawarden (1992) throughout each night, giving a photometric zero-point accuracy of $\Delta K < 0.05$ mag.

The entire data reduction and analysis from the raw data through the final number counts was done using two entirely independent data reduction packages, IRAF, and a package used in Soifer et al. (1994), which will be described in more detail elsewhere. The results, in terms of both photometry and the final counts, are in an excellent agreement. Here we present the results derived from the IRAF reductions. We have verified that our final stacked images have sensitivities that are quite

¹ Based on observations obtained at the W. M. Keck Observatory, which is operated jointly by the California Institute of Technology and the University of California.

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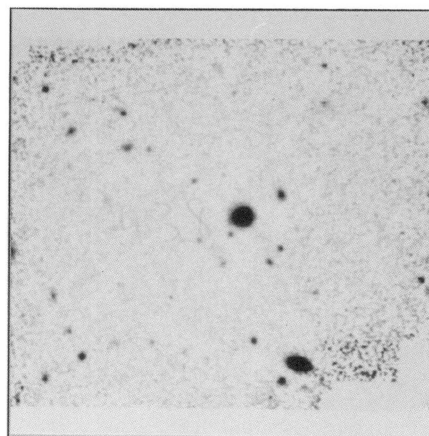
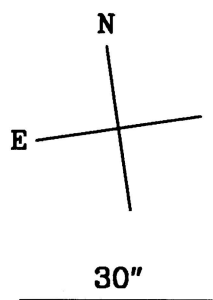
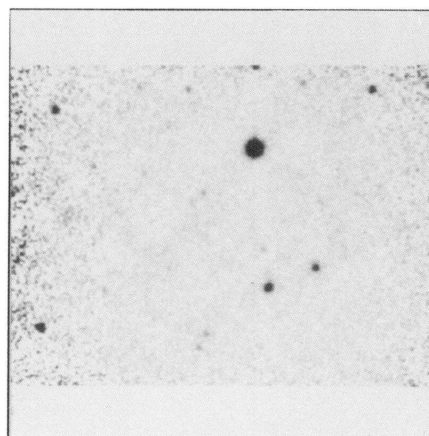
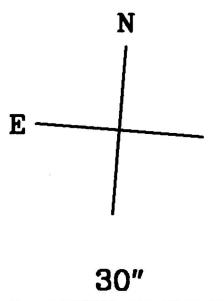
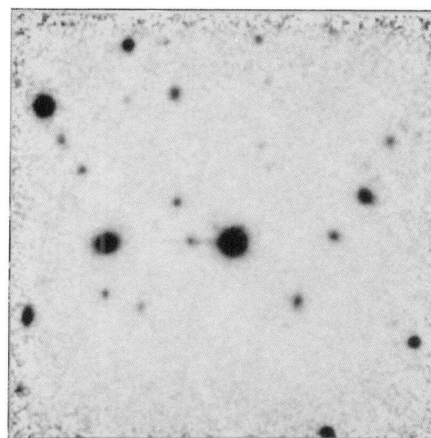
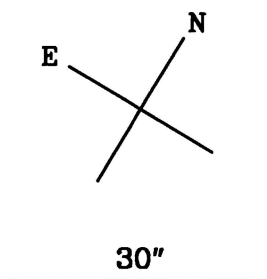
FP 1254+34 Field**FP 1454+34 Field****Hercules Field**

FIG. 1.—Co-added *K*-band images of the three fields, with the orientations and image scales as indicated. The coordinates, good to about 1'' of the brightest object (a stacking star or a galaxy) in each field are (equinox J2000): FP 1254 + 34: $\alpha = 12^{\text{h}}56^{\text{m}}30^{\text{s}}.6$, $\delta = +34^{\circ}11'17''$; FP 1454 + 34: $\alpha = 14^{\text{h}}56^{\text{m}}42^{\text{s}}.4$, $\delta = +33^{\circ}55'11''$; Hercules: $\alpha = 17^{\text{h}}22^{\text{m}}18^{\text{s}}.9$, $\delta = +49^{\circ}53'13''$.

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close to the theoretical estimate based on the sky flux of the original frames. In a typical 120 s exposure, we detect $\sim 8.2 \times 10^5 e \text{ pixel}^{-1}$, or $\sim 1.4 \times 10^8 e \text{ pixel}^{-1}$ for the stacked images. The estimated flat-fielding accuracy is less than 10^{-4} per pixel, smaller than the Poissonian noise, and thus not a major contributor to the photometric errors. The final images typically have fields of view of $55'' \times 55''$ due to the telescope shifts between frames, with the regions of approximately $40'' \times 40''$ with the best sensitivity. We have used FOCAS (Jarvis & Tyson 1981) to search our images for sources. Using simulations of Gaussian random noise field with faint sources added, we have optimized the isophotal area and sensitivity limits for our faint source detection while minimizing the false source detection; our typical isophotal limit is close to $\mu_K = 24 \text{ mag arcsec}^{-2}$ for all three fields.

Extensive Monte Carlo simulations have been performed in order to test our photometry and completeness. The details of these simulations and our photometric procedures will be presented elsewhere. We use isophotal magnitudes for sources brighter than $K = 21 \text{ mag}$, and $1''.5$ diameter aperture magnitudes for sources fainter than this level. Aperture corrections have been made to the total flux in $5''.4$ diameter apertures, as determined from bright stars. The total aperture correction ranged between 0.22 and 0.33 mag for the three fields. We note that our aperture corrections, while optimized for the fainter galaxies, will underestimate the brightness of extended galaxies ($K \leq 19 \text{ mag}$), but that use of aperture corrections based on the brightest galaxies are probably too large for normal galaxies at cosmological redshifts, and dwarfs at any redshift. The simulations also give typical random errors of $\sim 0.1 \text{ mag}$ at $K = 21.5 \text{ mag}$, which then increase monotonically to $\sim 0.4 \text{ mag}$ at $K = 23.5 \text{ mag}$.

We estimate the completeness from simulations on our images by adding five objects, which are rescaled bright objects from the field, with magnitudes between $K = 21$ to 25 mag , to random locations to the images. We have performed 500 such simulations for each of our three images. We are 90% complete at $K \approx 23 \text{ mag}$, and 50% complete at $K \approx 23.5 \text{ mag}$ for the three fields. We have corrected the source counts accordingly. These numbers have been tested by searching for spurious objects in 1000 synthesized Gaussian random noise fields equivalent to our limiting sensitivities, using the same algorithms. We detect a small number of false sources, mostly in the range $K = 23.5$ to 24 mag , and have used this number function of false detections to subtract from our number counts. This correction is ≈ 3.5 sources for the $K = 23\text{--}24 \text{ mag}$ bin, which is approximately a 20% correction, or ≈ 1.5 sources for the $K = 22.5\text{--}23.5 \text{ mag}$ bin, a 5% correction.

In summary, we reach the limiting magnitudes $K \sim 23.5\text{--}24 \text{ mag}$ and the 1σ noise limits of $K \approx 25.5 \text{ mag arcsec}^{-2}$, and $K \approx 25.2 \text{ mag per } 1''.5 \text{ aperture}$ ($\approx 2 \text{ FWHM}$). We note that the corresponding average sky surface brightness levels are $K \approx 13.35 \text{ mag arcsec}^{-2}$, and $K \approx 12.7 \text{ mag per } 1''.5 \text{ aperture}$. Thus, the data span a dynamical range of $\sim 10^5$ for our photometry apertures.

Optical observations were obtained at Palomar. The Hercules field was observed on the nights of UT 1992 May 29, June 8–10, August 2–3, and 1993 May 14, using the COSMIC imager on the 5 m Hale telescope. The total exposure times are 6000 s in Gunn g , 24,000 s in Gunn r , and 2300 s in Cousins I . The seeing FWHM on the final stacked images are $0''.90$, $0''.87$, and $1''.15$ in g , r , and I , respectively. In a $3''.0$ diameter apertures, we get 4.5σ detections at $r = 26.0 \text{ mag}$, where the photometric errors are $\Delta r \sim 0.15 \text{ mag}$, and a median error of $\Delta g \sim 0.25 \text{ mag}$ at $g = 26 \text{ mag}$. The other two fields were observed using the 4-Shooter imager on the Hale telescope, but not to this depth, and will be reobserved in the future.

3. DEEP GALAXY COUNTS AND COLORS

The final number counts, both raw and corrected, are listed in Table 1 for two separate binning patterns. The three fields are combined into our final number counts by weighting them by their area coverage. All sources were included in the number counts, with no attempts at identifying nor excising stars from our counts, since these faint galaxies have profiles that are not easily distinguished from the PSF in a low S/N situation. However, at these faint magnitudes, high Galactic latitudes, and small areas covered, there are very few faint stars expected in the infrared (Cowie et al. 1994), and any such contamination is expected to be small. We consider our number counts to be statistically unreliable for $K \leq 19.5 \text{ mag}$, since we detect only a few sources per field between $K = 18.5$ and 19.5 mag .

Our results are compared with the other deep K -band number counts in Figure 2. Our counts continue to rise, with no sign of a turnover, down to the limits of our data. In the magnitude interval between $K = 20$ and 24 mag , the fitted logarithmic slope is $d \log N/dm = 0.315 \pm 0.02$ (the Euclidean value is 0.6). We note that our number counts tend to be slightly lower than Cowie et al. (1994). This may be due to the differences in aperture corrections used; if we instead adopt their aperture correction procedure, then our number counts are brought into agreement with their work. Integrating over the power-law fits, we derive a cumulative surface density of $\sim 5 \times 10^5 \text{ galaxies deg}^{-2}$, or $\sim 2 \times 10^{10}$ over the entire sky, down to $K = 24 \text{ mag}$. The number counts from Soifer et al. (1994) appear to be significantly higher than those described in

TABLE 1
THE OBSERVED NUMBER COUNTS AND SURFACE DENSITIES

K (mag)	FP 1254		FP 1454		HERCULES		TOTAL	
	N_{raw}	$\log N_{\text{corr}}$ ($\text{deg}^{-2} \text{ mag}^{-1}$)	N_{raw}	$\log N_{\text{corr}}$ ($\text{deg}^{-2} \text{ mag}^{-1}$)	N_{raw}	$\log N_{\text{corr}}$ ($\text{deg}^{-2} \text{ mag}^{-1}$)	N_{raw}	$\log N_{\text{corr}}$ ($\text{deg}^{-2} \text{ mag}^{-1}$)
20.0.....	3	4.27 ± 0.41	1	3.88 ± 0.64	2	4.17 ± 0.41	6	4.15 ± 0.21
21.0.....	7	4.64 ± 0.19	2	4.18 ± 0.41	7	4.90 ± 0.19	16	4.57 ± 0.12
22.0.....	14	4.94 ± 0.13	7	4.73 ± 0.19	5	4.59 ± 0.24	26	4.79 ± 0.09
23.0.....	17	5.15 ± 0.12	13	5.19 ± 0.14	10	5.06 ± 0.16	40	5.13 ± 0.07
20.5.....	6	4.58 ± 0.21	3	4.36 ± 0.32	5	4.57 ± 0.24	14	4.52 ± 0.13
21.5.....	9	4.75 ± 0.17	2	4.18 ± 0.41	6	4.65 ± 0.21	17	4.60 ± 0.12
22.5.....	18	5.07 ± 0.11	8	4.80 ± 0.18	7	4.77 ± 0.19	33	4.91 ± 0.08
23.5.....	15	5.43 ± 0.13	19	5.48 ± 0.11	13	5.31 ± 0.14	47	5.37 ± 0.06

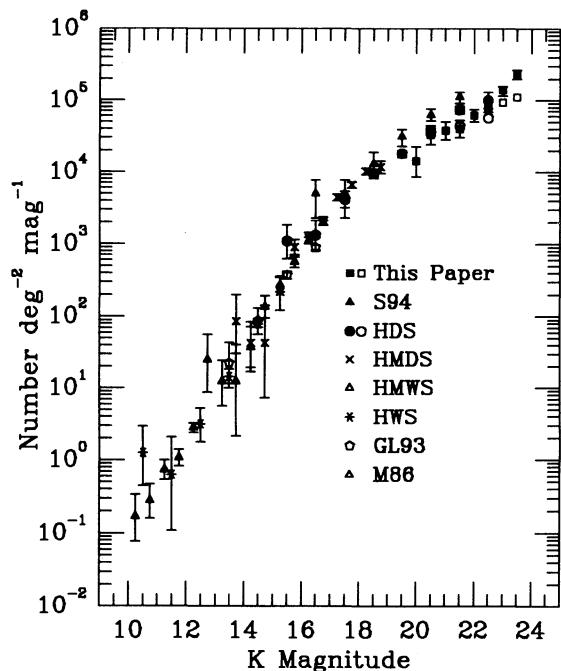


FIG. 2.—Extragalactic number counts in the K band. We have included the compilation of Gardner et al. (1993) for the following data: Soifer et al. (1994: S94); Gardner et al. (1993: HDS, HMDS, HMWS, and HWS); Glazebrook et al. (1993: GL93), and Mobasher, Ellis, & Sharples (1986: M86). The open symbols for this paper and HDS represent raw number counts, while the filled symbols for these two surveys represent the completeness-corrected counts.

this Letter and in Cowie et al., which cannot be explained by only a difference in photometric system. The counts of Soifer et al. were obtained by areas surrounding high-redshift target objects, and may reflect real overdensities due to clustering at large z , which in itself would be an exciting result.

We do see some field-to-field variations in our counts. Poisson noise and faint galaxy clustering both contribute to these. For a correlation function of the power-law form $w(\theta) = A_w \theta^{-\gamma}$ and a circular top-hat window function (chosen for simplicity) of angular radius θ_0 , the rms variation due to clustering is $f(\gamma)w(\theta_0)^{1/2}N$, where $f(\gamma) \sim 1$, and N is the mean number of galaxies per field. The median color in our sample is $(r - K) \sim 3$ mag, so we expect our $K \leq 23.5$ mag sample to have similar clustering to an $r \leq 26.5$ mag sample. Brainerd, Smail, & Mould (1994) measure $w(\theta) = A_w \theta^{-0.8}$ in a complete $r \leq 26$ mag sample, and show that $\log A_w \sim -0.3r_{\text{lim}}$, where r_{lim} is the limiting magnitude. Extrapolating their results to $r_{\text{lim}} = 26.5$ mag, we find that $A_w = 0.08$ (for θ in arcsec) for our sample. The area coverage of each field corresponds to that of a circle of radius $\theta_0 = 24''$, and $f(0.8) = 1.20$. With an average of $N = 32$ objects with $K \leq 23.5$ mag per field we expect an rms field-to-field variation due to clustering of 3.0 counts, while the Poisson rms variation is 5.7. This is consistent with the observed variations in our counts.

We compare the deep K-band galaxy counts with theoretical predictions in Figure 3. We use directly a no-evolution model from Koo, Gronwall, & Bruzual (1993), which is for an open universe with $q_0 = 0.05$, as well as models from Yoshii & Takahara (1988, hereafter YT), both with and without evolution, and for $q_0 = 0.02$ and 0.5. We normalize the YT models at $K = 15$ mag, where $\log N = 2.2$, which should bypass any problems with the local luminosity function normalization and

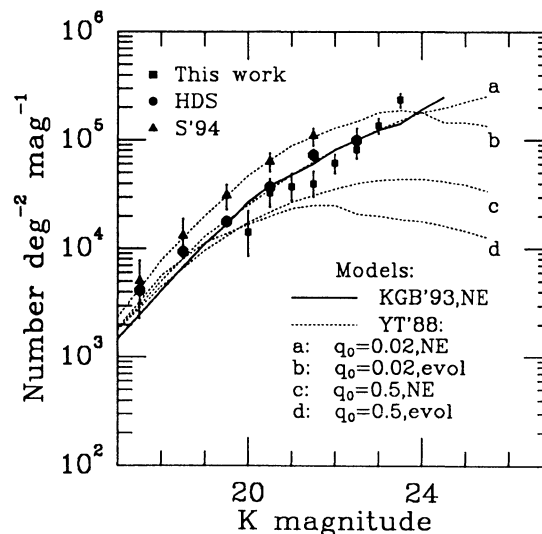


FIG. 3.—Zoom-in on the faint end of the number counts from the Hawaii deep survey (HDS), Soifer et al. (1994: S'94), and this work, with theoretical models superposed. The solid line is the no-evolution (NE) model which assumes a $q_0 = 0.05$ cosmology, from Koo et al. (1993). No normalization was applied to it. The dotted lines are models from Yoshii & Takahara (1988), for two values of the q_0 (0.02 or 0.5), with or without galaxy evolution (evol or NE). These were normalized to $\log N = 2.2$ at $K = 15$ mag.

the large-scale structure, while not being affected by the cosmological or evolution effects. The results are rather striking, and in contrast with the conclusions reached by Gardner et al. (1993): we find that the best fit to the data is provided by the no-evolution models in an open universe. The difference may be partly due to the normalization of the models, which Gardner et al. do at $K = 14.5$ mag. We find that the YT $q_0 = 0.5$ models underpredict the counts by a substantial amount. Obviously some galaxy evolution must be going on, but the good fit of the “no evolution” models implies only a weak evolution. This is fully consistent with the results from deep redshift surveys, which imply little evolution out to $z \sim 1$, and favor open universes, or even models with a positive cosmological constant. We note that Gardner et al. also plot a model for the B-band counts with a cosmological constant term, from Fukugita et al. (1990), and claim that it is excluded by their data. It is not obvious that this model can be easily compared to the K-band counts, and, lacking a better model, the issue of the cosmological constant is still very much open. In general, it is probably premature to make any strong statements about the global geometry from these counts: there are too many adjustable parameters in the models, and the redshift distribution may be essential in discriminating between different cosmological (and evolution) models. All that we can say is that there is no real support for the $\Omega_0 = 1$ cosmology, from the K-band galaxy counts alone.

In order to compute the optical-to-IR colors, we have smoothed the three final images to the resolution of the optical images and measured the flux in $3''$ diameter apertures for all sources detected at $\geq 5 \sigma_{\text{rms}}$ in the K band. The σ_{rms} is the noise in a $1.5''$ diameter circular aperture, resulting in a 5σ limiting magnitude of approximately $K = 23.2$ at the center of the three fields. Color-magnitude diagrams for all sources in the Hercules field are shown in Figure 4. Error bars along the K-magnitude axis are the addition in quadrature of the zero-point error (0.05 mag), the photometric error for the aperture,

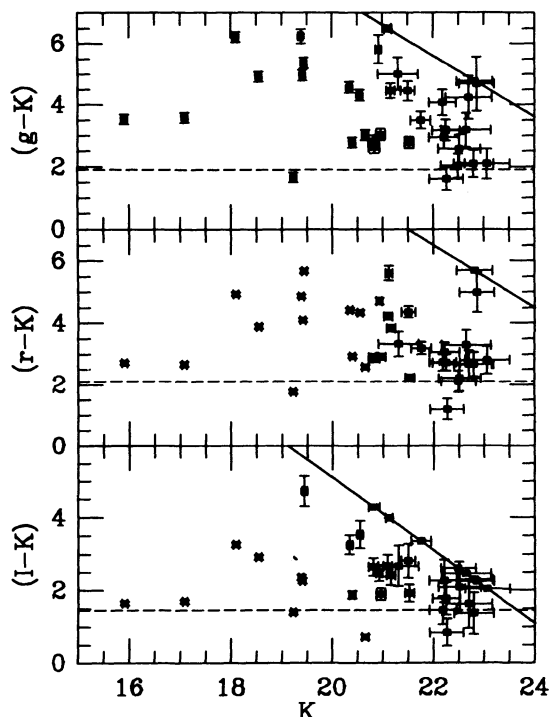


FIG. 4.—Galaxy colors as a function of the K -band magnitude, for all objects detected in the Hercules field. The diagonal lines indicate the limits of the optical data; points on them are lower limits. The dashed lines correspond to objects with flat F_{ν} spectra.

and the recovered magnitude scatter error determined from the completeness simulations described above. The latter varied from 0.05 mag at $K = 21$ to 0.4 mag at $K = 23$ mag, and was the dominant source of uncertainty, for the faint objects. Error bars along the color axis are the addition in quadrature of the K -band magnitude error and the optical band magnitude error, including both the zero-point and the random errors. The diagonal lines represent the 1σ detection limit in a $3''$ aperture, for the grI data. For the Hercules field, the limits are 27.6 mag in g , 28.5 mag in r , and 25.1 mag in i .

We see at most a weak bluing trend going toward fainter K magnitudes. We do see some very red galaxies, with $(r - K) \gtrsim 5$ mag, or $(i - K) \gtrsim 5$ mag. Their frequency seems to be consistent with what was found by Cowie et al. (1994) at the comparable magnitude levels, viz., ≈ 1 – 2 per arcmin 2 . Following Elston, Rieke, & Rieke (1991), they may well be ellipticals at $z \sim 1$. Unfortunately, our present optical data do not reach deep enough to make any statements about the possible extremely red galaxies (see Hu & Ridgway 1994). It is also in principle possible that some of the extremely red, faint objects are low-mass M dwarfs, or even brown dwarfs. Simple esti-

mates suggest that at most one or two such stars would be found in our data, even if the entire dark halo is composed of such objects. However, we need a much larger area coverage before this possibility can be addressed meaningfully. Our preliminary data on colors in the other two fields are consistent with what we see in the Hercules field, even though our present CCD images do not reach as deep. A more complete analysis of the colors will be presented in a future paper, when deeper CCD images are obtained.

We also note that the estimated line fluxes from protogalaxies powered by star formation rates of the order of a $100 M_{\odot} \text{ yr}^{-1}$ at redshifts $z \sim 2$ – 5 are expected in the range of $\sim 10^{-16 \pm 1} \text{ ergs cm}^{-2} \text{ s}^{-1}$ (Thompson, Djorgovski, & Beckwith 1994). Assuming appropriate equivalent widths, this corresponds to broad-band magnitudes $K \sim 22 \pm 2$ mag. There is thus an intriguing possibility that such a population of protogalaxies may be detectable at the magnitude levels that we are now reaching. The surface density of such objects is expected to be of the order of ~ 1 – 10 per arcmin 2 , and thus it is possible that one or more of the faintest objects we detect is a young galaxy.

4. CONCLUDING REMARKS

We have presented the deepest galaxy counts to date in the K band, reaching the limiting magnitudes $K \sim 23.5$ – 24 mag. An L_{*} galaxy ($M_K \approx -25$ for $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$) at this flux level could be detected out to $z \sim 3$ – 4 , or even beyond, depending on the cosmology and evolution. We are thus clearly probing a cosmologically relevant regime.

The counts continue to rise with a power-law slope of ≈ 0.31 , with no evidence for a turnover down to the limits of our data. The counts can be well fitted with models incorporating no (or very little) evolution, and a low value of Ω_0 ; high- Ω_0 cosmologies are not required by the data. However, we feel that more carefully constructed models, including the effects of merging, are needed before any strong statements about the global geometry can be made from the counts.

The W. M. Keck Observatory is a scientific partnership between the California Institute of Technology and the University of California. It was made possible by the generous and visionary gift of the W. M. Keck foundation, and the support of its president, Howard Keck. It is a pleasure to thank the observing assistants T. Chleminiak and B. Schaefer for the expert work during our observing runs, J. R. Mould for doing the Palomar imaging of the Hercules field, and M. Wilber for the help with the models. This work was supported in part by the NSF PYI award AST-9157412 to S. D., the NSF grant AST-9223370 to R. D. B., grants from the NSF and NASA to B. T. S., G. N., and K. M., a NATO Advanced Fellowship to I. S., and an NSF graduate fellowship to D. W. H.

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